ſides; and the clear water only is from the ſurface of the baſon let out into the pits.

If there be any thing, wherein I may further ſatisfy your lordſhip's inquiries in this or any other matter, your commands ſhall moſt chearfully be obey'd by

Your lordſhip's much obliged, and

moſt obedient humble ſervant,

William Henry.

XIII. *The Conſtruction of the* logarithmic Lines *on the* Gunter's Scale; *by Mr.* John Robertſon, *F. R. S.*

Read June 18, 1752.

HAVING lately had occaſion to treat on the conſtruction of the Gunter's ſcale, I ſearched ſeveral books, wherein I ſuſpected were contained the reaſons of the common methods of laying down the logarithmic lines uſually put on thoſe ſcales: but not finding, either from my own ſearch, or that of my friends, any ſatisfactory account of this matter, I drew up the following paper, to be laid before the Royal Society.

The Gunter's ſcale * is an inſtrument almoſt univerſally known, and amply deſcribed by many writers; therefore

* So called from its inventor Mr. Edmund Gunter, aſtronomy-profeſſor in Greſham-College, from March 6, 1619, till his death, Dec. 10, 1626.

therefore I ſhall not take up your time in uſeleſs repetitions. but only ſhew, on what principles the diviſions of the logarithmic ſines, tangents, and verſed ſines, are uſually protracted.

The line of numbers on theſe ſcales conſiſts of two equal lengths, commonly called two radii; the firſt containing the logarithms of numbers from 10 to 100; and in the ſecond are inſerted thoſe between 100 and 1000, or ſuch of them, as can conveniently be introduced.

Theſe diviſions are taken from a ſcale of equal parts; ſuch, that 100 make the length of one radius; and from this ſcale, the diviſions for the ſines, tangents, and verſed ſines, are alſo taken. Now, from this conſtruction of the line of numbers, it is plain, that, as the numbers in one radius exceed thoſe in the other, by one place in the ſcale of numeration; therefore the difference of their indices muſt alſo be unity: ſo that ſuch numbers only, whoſe index differs by 1, can be eſtimated in a length of two radii: but, in a length of three radii, numbers, whoſe indices differ by 2, may be read; and a difference of 3 may be reckon'd in a length of 4 radii, *&c.* The tables of logarithmic ſines, tangents, ſecants, and verſed ſines, are generally computed for a circle, whoſe radius is 10,000,000: therefore,

	°	′	″		°	′	″
In the ſines, the index 9 belongs to all between	90	0	0	and	5	44	36
The index 8	5	44	36	and	0	34	23
The index 7 to all between	0	34	23	and	0	3	27
6	0	3	27	and	0	0	21

&c.

In the tangents, the index 9 belongs to all between 45° 0′ and 5° 42′ 10″
And the indices, 8, 7, 6, *&c.* fall as in the ſines.

	°	′		°	′
In the verſed ſines, the index 10 belongs to all between	180	0	and	90	0
9	90	0	and	25	51
8	25	51	and	8	7
7	8	7	and	2	34
6	2	4	and	0	45

&c.

Now, as the length of the Gunter's ſcale admits of no more than two radii, or of ſuch numbers only, whoſe index differs by unity; therefore, within this length, no more of the ſines, tangents, or verſed ſines, can be introduced, than thoſe, whoſe index differs by unity: And as not only the greateſt number among the ſines and tangents, but alſo thoſe more generally wanted, have the indices 9 and 8 differing by unity; therefore all the ſines from 90° to 0° 34′, and all the tangents from 45° to 0° 34′, are thoſe only, which are put on theſe ſcales; the diviſions anſwering to the leſſer ſines and tangents being omitted for want of room. And this is the reaſon, why the ſine of 90°, and the tangent of 45°, are limited by the ſame termination as the ſecond radius on the line of numbers.

To conſtruct the line of logarithmic ſines.

From the ſcale of equal parts, take the numbers expreſſing the arithmetical complements of the log. ſines

ſines of the ſucceſſive degrees, and parts of degrees, intended to be put on the ſcale, deſcending orderly from 90° : then theſe diſtances ſucceſſively laid from the mark repreſenting 90° at the right-hand end of the ſcale, will give the ſeveral diviſions of a ſcale of logarithmic ſines.

For, the ends of any ſcale being aſſigned, the progreſſive diviſions of that ſcale are laid thereon from that end, which repreſents the beginning of the progreſſion: or, the ſame diviſions may be laid from the other end, by taking the complements of the terms to the whole length of the ſcale:

Conſequently the arithmetical complements of the ſines are to be laid from the diviſion repreſenting 90 degrees.

To conſtruct the line of logarithmic tangents.

Theſe are laid down in the ſame manner, and for the ſame reaſons, that the ſines were; the tangent of 45° ſtanding againſt the ſine of 90°.

The diviſions for the tangents above 45°, are reckoned on the ſame line from 45° towards the left-hand; or any tangent and its co-tangent are expreſſed by the ſame diviſion.

Thus one mark ſerves for 40° and 50°; and the diviſion at 30° ſerves alſo for 60°; that at 20° ſerves for 70°, *&c.* and the like is to be underſtood of the intermediate diviſions.

For, as the tangent of an arc, is to radius;
So is radius, to the co-tangent of that arc.

Therefore the tangent is equal to the ſquare of radius divided by the co-tangent.

And the co-tangent is equal to the ſquare of radius divided by the tangent.

Now the radius being unity, its ſquare is alſo unity.

Therefore the tangent and co-tangent of any arc are the reciprocals one of the other.

But the reciprocals of numbers are correlatives to the arithmetical complements of their logarithms.

Therefore the logarithms of a tangent and its co-tangent are arithmetical complements one of the other; and conſequently will fall at equal diſtances from 45 degrees.

Therefore, in the line of logarithmic tangents, the diviſions to degrees under 45 ſerve alſo for thoſe above; both being equally diſtant from 45 degrees.

To conſtruct the line of logarithmic verſed ſines.

As the greateſt number of degrees will fall within the limits of the ſcale by beginning at 180°; therefore the termination of this line is at 180°, which is put againſt 90° on the ſines: and altho' the numbers annexed to the diviſions increaſe in the order from right to left, yet they are only the ſupplements of the verſed ſines themſelves.

Now ſubtract the logarithmic verſed ſines of ſuch degrees and parts of degrees as are intended to be put on the ſcale, from the logarithm verſed ſine of 180°; then the remainders taken from the foreſaid ſcale of equal parts, and laid ſucceſſively from the termination of this line, will give the ſeveral diviſions ſought.

The following table to every 10 degrees was conſtructed in the foregoing manner, and are the numbers

to

to be taken from the ſcale of equal parts, for the degrees they ſtand againſt.

Degrees	Supplements of Verſed Sines	Degrees	Supplements of Verſed Sines	Degrees	Supplements of Verſed Sines
180	0,00000	120	0,12494	60	0,60206
170	0,00331	110	0,17327	50	0,74810
160	0,01330	100	0,23149	40	0,93190
150	0,03011	90	0,30103	30	1,17401
140	0,05403	80	0,38387	20	1,52066
130	0,08545	70	0,48282	10	2,21941

From this table it appears, that the leaſt verſed ſine, which can be introduced within the length of a double radius, falls between 10° and 20°, where the index changes from 1 to 2; which will happen about 11° 28′.

If a table of logarithm verſed ſines to 180° are wanting, they are eaſily made by the following rule.

> Take the logarithm ſine of 30 degrees from twice the logarithm ſine of (N) any number of degrees; the remainder is the logarithm verſed ſine of ($2N$, or) twice thoſe degrees."

For it is a well-known goniometrical property, that the ſine of any arc (A) is a mean proportional between radius (R) and half the verſed ſine of twice that arc.

Therefore, putting v for the verſed ſine, and s for the ſine;

The

The $v\,2\,A = \left(\frac{2\,ss\,A}{R} = ss\,A \times \frac{2}{R} = ss\,A \times \frac{2}{10} = \right)$ $ss\,A \times \frac{1}{5}$; radius being 10.

Or the log. $v\,2\,A = 2$ log. $s\,A$ — log. 5.

But when radius is 10, the fine of 30° is 5.

Therefore the log. $v\,2\,A = 2$ log. $s\,A$ — log. fine of 30°.

Moft of the writers on this fubject give the following rule for laying down the divifions of this line :

> From the line of logarithmic fines, take the diftance between 90° and any arc; that diftance being twice repeated, from the termination of the line of verfed fines, will give the divifion for twice the complement of that arc."

Thus the diftance between 90° and 20° on the fines twice repeated, gives the verfed fine of 140°; or twice 70°, the complement of 20°. For the divifions, to be laid on this line, are the differences between the logarithm verfed fine of 180°, and the logarithm verfed fines of the fucceffive arcs.

Now the difference between the logarithm verfed fines of 180°, and of any arc $2\,A$, is log. ver. fine 180 — 2 log. fin. A + log. fin. of 30°.

Or, 10,30103 + 9,69897 — twice log. fin. of A.

Or, 20,00000 — twice logarithm fine of A.

Or the arithmetical complement of twice logarithm fine of A.

That is, the difference between the logarithm verfed fine of 180°, and the logarithm verfed fine of any arc, is equal to double the arithmetical complement of of the logarithm ſine of half that arc, rejecting the indices.

But, as theſe differences give the diviſions to the ſupplements of the real verſed ſines; therefore the arithmetical complement of the logarithm ſine of any arc being doubled, will give the diſtance of the diviſion for the ſupplement of twice that arc on the line of verſed ſines.

Thus, for 70°, the logarithm ſine is	9,97299
The arithmetical complement is	0,02701
Its double is	0,05402

Which is the number in the foregoing table ſtanding againſt 140°, and is the ſupplement verſed ſine of twice 70 degrees.

Now, as the arithmetical complement of the log. ſines of arcs, are the diſtances on the line of ſines between 90°, and the diviſions to thoſe arcs; therefore the diſtances between 90° and any arc, being twice repeated, will give the diviſion of the ſupplemental verſed ſine to twice the co-ſine of that arc.

XIV. *A Letter from Mr.* John Dollond *to Mr.* James Short, *F. R. S. concerning an Improvement of* refracting Teleſcopes.

SIR.

Read March 1, 1753.

IT is well known, that the perfection of refracting teleſcopes is very much limited by the aberration of the rays of light from the geometrical focus; which ariſes from two very different cauſes; that is, from different degrees